

Math 590
Mathematics of Fermat's Last Theorem
Fall 2010 Notes

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INTRODUCTION

This set of notes is designed for a graduate level number theory course at Nicholls State University entitled “Mathematics of Fermat’s Last Theorem”. They are suitable for a one semester course. The student should have familiarity with logic, set theory, and have theorem-proving experience. A previous course in number theory, while not essential, would also be helpful.

In 1637, Pierre Fermat was reading about the Pythagorean Theorem (how a square could be divided into two other squares) and remarked in the margin of the book he was reading:

“However, it is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general any power higher than the second into two powers of like degree. I have discovered a truly remarkable proof which this margin is too small to contain.”

As often happens, this innocuous comment instigated 350 years of research. Mathematicians attempted to find Fermat’s proof (for he died before revealing it) or to produce one of their own. Most failed. It was eventually assumed that Fermat was either teasing his colleagues (and did not really have such a proof) or more likely his proof was incorrect. By the late 20th century, British mathematician Andrew Wiles successfully proved Fermat’s Last Theorem using some of the most cutting edge algebraic and analytic number theory of his time.

We will begin by discussing the origin of the famous “last” theorem and its history. We will cover various special cases that were proved early on as well as problems that arose. Finally, we will walk through the eventual proof by Andrew Wiles covering some advanced mathematics where we can.

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